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Rotating ferro-nanofluid over stretching plate under the effect of hall current and joule heating

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ABSTRACT

The behavior of boundary layer over a stretching plate filled with ferromagnetic Fe_3O_4 nanoparticles and subjected to magnetic field with hall current, joule heating and nonlinear thermal radiation has been investigated. The modeling based on nonlinear partial differential equations due to continuity, momentum and heat equations, these equations transformed to a system of nonlinear ordinary differential equations using similarity transformation technique then solved numerically. The effect of hall current, joule heating and thermal radiation on the physical quantities such as surface shear stress and heat flux have been investigated and discussed. Moreover, the velocities and temperature profiles of the boundary layer under the influence of the presented external forces plotted and discussed.

1. Introduction

FerroNanofluids are colloidal liquids which made of carrier fluids such as (water, oil…) and nanoparticles size that suspended by Brownian motion. These fluids become strongly magnetize in the presence of magnetic field that help it to find applications in micro-fluidic devices and microelectromechanical systems due to its high thermal conductivity comparing to the other regular fluids. Ferrofluids has very different applications in industry and engineering applications such as, forming a seals around the spinning drive shafts in hard disks, miniature microscale devices, loudspeakers to remove heat from the voice coil, and to passively damp the movement of the cone, semi-active dampers in mechanical and aerospace applications. These fluids not stop at the engineering applications but also extend for medical applications; almost all applications in medicine exploit the extreme relative size difference between magnetic nanoparticles and living cells. Most important of these applications are an experimental cancer treatment called targeted magnetic hyperthermia; enhance the contrast agent in magnetic resonance imaging and magnetic separation of cells.

In the thermodynamic field, Tangthieng et al. [1] have studied the heat transfer enhancement in ferrofluids subjected to steady magnetic fields. Jue [2] presented a numerical analysis of combined magnetic gradient and thermal buoyancy induced cavity ferrofluid. Sheikholeslami et al. [3] investigated the influence of an external magnetic field on ferrofluid flow and heat transfer in a semi annulus enclosure with sinusoidal hot wall. Generally, using nanoparticles within pure fluids to improve its thermal conductivity suggested by choi et al. [4] they concluded that the invention of nanofluids presents new challenges and opportunities for thermal scientists and engineers. In fact, the enhancement of the fluids thermal conductivity opened a new area for many researchers to study the effect of nanoparticles on the characteristics of heat transfer. Freidoonimehr et al. [5] presented a numerical investigation of MHD unsteady free convection flow of nanofluids past a vertical surface. Abolbashari et al. [6] analyzed analytically the unsteady MHD boundary layer over a porous stretching surface taking the effect of entropy generation due to conduction effect, fluid friction irreversibility, and joule dissipation irreversibility. Moraveji et al. [7] studied the natural convection heat transfer in rectangular cavities with an inside oval-shaped heat source filled with Fe_3O_4/water nanofluid. Malvandi et al. [8] presented a theoretical study to investigate the laminar flow and convective heat transfer of water/alumina nanofluid inside a parallel-plate channel under the effect of a uniform magnetic field. Selimefendigil et al. [9] analyzed numerically the heat transfer enhancement and fluid flow characteristics of a rotating cylinder under the effect of magnetic dipole. Sheikholeslami et al. [10] studied the effect of non-uniform magnetic field of Fe_3O_4–water nanofluid on the heat transfer in a lid driven semi annulus enclosure. In the case of strong magnetic field, some molecules that carry electrical charges may veer off from its path under the influence of Hall current force or Lorentz force. The effect of hall current on the boundary layer behavior have been studied by Abdel-wahed et al. [11,12] they investigated the effect of hall current with joule heating on MHD boundary layer due to nanofluid over a rotating disk. Chauhan et al. [13] studied the heat transfer effects on rotating MHD couette flow in a channel partially filled by a porous medium with hall current. Pandurangan et al. [14] studied the hall current effects on

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Nomenclature

a	Constant parameter (t^{-1})
u, v, w	Linear velocities (m/s)
T	Temperature (K)
q_r	Radiative heat flux (W/m^2)
B_0	Magnetic flux density (kg/s^2A)
k	Thermal conductivity (W/mK)
C_p	Specific heat ($J/kg\ K$)
C_f	Skin friction coefficient

Greek symbols

ϕ	Nanoparticles volume fraction
Ω	Angular velocity (rad/s)
ρ	Fluid density (kg/m^3)
μ	Dynamic viscosity (Ns/m^2)
ν	Kinematic viscosity (m^2/s)
σ	Electrical conductivity of fluid (S/m)
σ^*	Stefen-Boltzman constant ($1.3806488\times10^{-23} m^2kg/s^2K$)
α^*	Mean absorption coefficient

Dimensionless parameters

$\lambda = \frac{\Omega}{a}$	Rotation parameter
$P_r = \frac{\nu_f\left(\rho C_p\right)_f}{k_f}$	Prandtl number
$M = \frac{\sigma_f {B_0}^2}{a\rho_f}$	Hartman number
$\theta_w = \frac{T_w}{T_\infty}$	Temperature ratio
$E_c = \frac{u^2\rho_f}{\left(\rho C_p\right)_f (T_w - T_\infty)}$	Eckert Number
$Rd = \frac{4T_\infty^3}{k_{nf}\alpha^*}$	Radiation parameter

Subscripts

f	Fluid phase
nf	Nano-fluid
s	Solid particles
w	Condition of the wall
∞	Ambient condition

a rotating MHD flow of an exponentially accelerated horizontal plate.

On the other hand, many researchers have investigated the effect of thermal radiation on the thermal boundary layer over vertical/horizontal surfaces. Due to simplification of the energy equation, the term T^4 expressed as a linear function of temperature using Taylor series by neglecting the higher order terms. Using this assumption, Elbashbeshy et al. [15] investigated the effect the linear thermal radiation and the heat generation on the mechanical properties of a continuous moving cylinder during the cooling using nanofluid. Rashidi et al. [16] studied analytically the effect of linear thermal radiation and suction/injection process on the MHD viscoelastic fluid flow over a porous wedge. Elbashbeshy et al. [17] and Abdel-wahed et al. [18] have investigated the influence of heat generation and linear thermal radiation on the heat flux from a moving surface with variable thickness. Studying the effect of thermal radiation as a nonlinear term without simplification have been investigated by Pantokratoras et al. [19], they studied the Blasius flow with non-linear Rosseland thermal radiation. Abdel-wahed et al. [20,21] extended the work of Pantokratoras to study the effect of nonlinear Rosseland thermal radiation and magnetic field on the boundary layer behavior over a moving surface with variable thickness. Mustafa et al. [22] studied the rotating magnetite water nanofluid over a stretching surface under the influence of nonlinear thermal radiation.

Due to the practical applications of the rotating flows, it has more attention of many researchers. Rajeswari et al. [23] and Nazar et al. [24] they studied the effect of unsteady rotating flow over a stretching surface on the physical properties of the boundary layer. Zaimi et al. [25] studied the behavior of the boundary layer over a stretching surface in a rotating viscoelastic fluid. This work deals to extend the model of Mustafa et al. [22] to study the effect of hall current with joule heating and nonlinear thermal radiation on the boundary layer behavior due to rotating fluid consists of ferromagnetic Fe_3O_4 nanoparticles over a stretching plate.

2. Formulation of the problem

Consider steady-laminar-incompressible rotating fluid with constant angular velocity Ω containing an electrically conducting Fe_3O_4 Ferro-nanoparticles over a stretching plate moving with uniform velocity $U_w = ax$. Uniform magnetic fields of strength B_0 with hall current and nonlinear thermal radiation with heat flux $q_r = -\left(\frac{4\sigma^*}{3\alpha^*}\right)\left(\frac{\partial T^4}{\partial z}\right)$ are applied normal to the plate. The surface has uniform temperature T_w while temperature far away from the surface is T_∞ (see Fig. 1).

The equations describing the model are [22,25]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\rho_{nf}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - 2\Omega v\right) = \mu_{nf}\left(\frac{\partial^2 u}{\partial z^2}\right) - \frac{(\sigma_{nf}{B_0}^2)}{(1+m^2)}(u - mv) \tag{2}$$

$$\rho_{nf}\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + 2\Omega u\right) = \mu_{nf}\left(\frac{\partial^2 v}{\partial z^2}\right) - \frac{(\sigma_{nf}{B_0}^2)}{(1+m^2)}(v + mu) \tag{3}$$

$$(\rho Cp)_{nf}\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}\right) = k_{nf}\left(\frac{\partial^2 T}{\partial z^2}\right) + \frac{\partial^2 T^4}{\partial z^2}\left(\frac{4}{3}\frac{\sigma^*}{\alpha^*}\right) + \sigma_{nf}{B_0}^2(u^2 + v^2) \tag{4}$$

where ρ_{nf} is the Nanofluid density, μ_{nf} is Nanofluid viscosity, σ_{nf} is the electrical conductivity of Nanofluid, B_0 is the externally imposed magnetic field in z-direction, m is the Hall current parameter

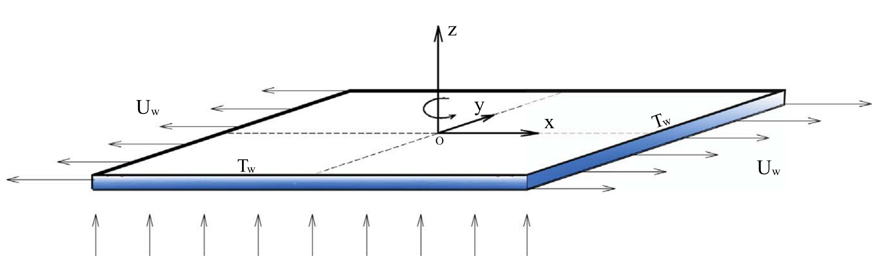

Fig. 1. Physical model and coordinate system.

($m=\tau_e\omega_e$); where τ_e is the electron collusion time and ω_e is cyclotron frequency, k_{nf} is the thermal conductivity, $(\rho C_p)_{nf}$ is the specific heat capacity, σ^* is the Stefen-Boltzman constant and α^* is mean absorption coefficient.

The boundary conditions for the present model are:

$$u = U_w, \quad v=0, \quad w=0 \quad , \quad T = T_w at z=0 \tag{5}$$

$$u\to 0, \quad v\to 0, \quad T = T_\infty, \quad as z\to\infty \tag{6}$$

The properties of Nanofluid defined as follows [17,22]

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}} \quad , \quad \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s \quad ,$$

$$(\rho C_p)_{nf} = (\rho C_p)_f(1-\phi) + \phi(\rho C_p)_s, \; K_{nf} = k_f\left(\frac{(k_s+2k_f)-2\phi(k_f-2k_s)}{(k_s+2k_f) + \phi(k_f - k_s)}\right)$$

$$\sigma_{nf} = \sigma_f\left(1+\frac{3\phi\left(\frac{\sigma_s}{\sigma_f}-1\right)}{\left(\frac{\sigma_s}{\sigma_f}+2\right) - \phi\left(\frac{\sigma_s}{\sigma_f}-1\right)}\right)$$

Here ϕ is the solid volume fraction, subscript s is for Nano-solid-particles, and subscript f is for base fluid.

The thermo-physical properties of water and the elements Fe_4O_3 shown in Table 1.

3. Similarity transformations

In this section, a similarity solution for Eqs. (1)–(4) with the boundary conditions (5) and (6) suggested in the following form

$$\left\{\begin{matrix} u = axf'(\eta) \quad , \quad v = axg(\eta), \quad w=-\sqrt{a\nu_f}f(\eta), \\ T = T_\infty+\Delta T\theta(\eta) \quad , \quad \eta = \sqrt{\frac{a}{\nu_f}}z \end{matrix}\right\} \tag{7}$$

Where η is the similarity variable. Substitute Eq. (7) into Eqs. (2)–(4), the following system of non-linear ordinary differential equations is obtained:

$$\left(\frac{A_4}{A_1}\right)f''' - f'^2 + ff''+2\lambda g - \left(\frac{A_5}{A_1}\right)\frac{M}{(1+m^2)}(f' - mg) = 0 \tag{8}$$

$$\left(\frac{A_4}{A_1}\right)g'' + fg' - f'g-2\lambda f'-\left(\frac{A_5}{A_1}\right)\frac{M}{(1+m^2)}(g + mf') = 0 \tag{9}$$

$$\theta'' + P_r\left(\frac{A_2}{A_3}\right)f\theta' + A_3Rd[3(\theta_w-1)(1 + (\theta_w-1)\theta)]^2\theta'^2 + A_3Rd[(1 + (\theta_w-1)\theta)]^3$$
$$\theta'' + P_rEcM\left(\frac{A_5}{A_3}\right)(f'^2 + g^2)=0 \tag{10}$$

where the prime denotes differentiation with respect to η *and*

$$A_1 = \frac{\rho_{nf}}{\rho_f} \quad , \quad A_2 = \frac{(\rho C_p)_{nf}}{(\rho C_p)_f}, \quad A_3 = \frac{k_{nf}}{k_f}, \quad A_4 = \frac{\mu_{nf}}{\mu_f}, \quad and A_5 = \frac{\sigma_{nf}}{\sigma_f}$$

The transformed boundary conditions of the problem are:

$$f(0)=0, \quad f'(0)=1 \quad , \quad g(0)=0, \quad \theta(0)=1 \tag{11}$$

$$f'(\infty)=0, \quad g(\infty)=0, \quad \theta(\infty)=0, \tag{12}$$

Where $\lambda = \frac{\Omega}{a}$ is rotation parameter, $P_r = \frac{\nu_f(\rho C_p)_f}{k_f}$ is Prandtl number, $M = \frac{\sigma_f B_0{}^2}{a\rho_f}$ is Hartman number, $\theta_w = \frac{T_w}{T_\infty}$ is the temperature ratio, $E_c = \frac{u^2\rho_f}{(\rho C_p)_f(T_w - T_\infty)}$ is Eckert Number, and $Rd = \frac{4\sigma^* T_\infty^3}{k_{nf}\alpha^*}$ is Radiation parameter.

4. Solutions and numerical results

The Eqs. (8–10) with the boundary conditions (11) and (12) solved as an initial value problem with unknown values of $f''(0), g'(0) and \theta'(0)$. In this process, a suitable finite value $\eta\to\infty$, say η_∞ should be suggested. The suitable guess values for $f''(0), g'(0) and \theta'(0)$ are chosen and then integration is carried out and compare the calculated values for $f''(0), g'(0) and \theta'(0)$. at $\eta = \eta_{max}$ with the given boundary conditions $f'(\eta_\infty)=0, g'(\eta_\infty)=0 and \theta'(\eta_\infty)=0$ and adjust the estimated values. By taking a series of values for $f''(0), g'(0) and \theta'(0)$ and apply the fourth order classical Rung–Kutta method with step-size $\Delta\eta=0.01$. The above procedure is repeated until we get the converged results within a tolerance limit of 10^{-6}.

To validate the accuracy of the results obtained by this method, the velocity gradients along x and y-axes and the values of $\frac{Nu}{\sqrt{Re}}$ considered in Tables 2 and 3 and compared with the numerical solution, which reported in Mustafa et al. [22].

The physical quantities of interest are the local skin friction coefficients C_{fx} & C_{fy} and the local Nusselt number Nu. Physically, these quantities indicate to surface shear stress τ_{fx} , τ_{fy} and the surface heat flux q_w.

$$\tau_{fx} = \mu_{nf}\left[\frac{\partial u}{\partial z}\right]_{z=0} \quad , \quad \tau_{fy} = \mu_{nf}\left[\frac{\partial v}{\partial z}\right]_{z=0}, \quad q_w = \left[q_r - k_{nf}\frac{\partial T}{\partial z}\right]_{z=0} \tag{13}$$

The final dimensionless forms are:

$$\sqrt{Re}\,C_{fx} = A_4 \; f''(0), \quad \sqrt{Re}\,C_{fy} = A_4 \; g'(0),$$
$$\frac{Nu}{\sqrt{Re}}=-(A_3 \; + Rd\theta_w^3)\theta'(0) \tag{14}$$

5. Discussion

This work presented a simulation model for rotating Ferro-nano-fluid consists of water as a base fluid with prandtl number Pr=6.2 and Fe_3O_4 nanoparticles over a stretching plate. The influence of magnetic field with a hall current, joule heating and the nonlinear thermal radiation on the momentum and thermal boundary layer investigated through a set of Figs. 2–14. it is worth mentioning that, the velocity along x-axis dubbed as longitudinal velocity and along y-axis dubbed as transversal velocity. Moreover, the effect of all embedded parameter on the rate of heat transfer and the surface shear stress presented through Tables 4, 5 and 6. Comment and analysis on the derived results were as follows:

- **The effect of magnetic field and hall current**

  Hartman number (M) is the dimensionless parameter that indicates to the effect of magnetic field. Figs. 2, 3, and 4 depict the impact of magnetic field for weak and strong magnetic strength on the boundary layer velocities. It is clear that, as Harman number raised, the velocities in both directions reduced. It worth mentioning that, the transversal velocity plotted at the negative zone of y-axis due to the flowing of the fluid in the opposite direction. Physically, applying magnetic field produces a boundary layer thinner, which reduce the ability of motion of the fluid in each direction. On the

Table 1
Thermo-physical properties of water and the elements Fe_3O_4 [22].

Properties	Fluid (water)	Fe_3O_4
$Cp(j/kgK)$	4179	670
$\rho(kg/m^3)$	997.1	5180
$k(W/mK)$	0.613	9.7
$\sigma(\Omega.\ m^{-1})$	0.05	25000

Table 2
Comparison of present results with previous study at M=m=ϕ = λ = 0..

λ	Mustafa et al. [22]		Present results	
	$f''(0)$	$g'(0)$	$f''(0)$	$g'(0)$
0.50	-1.13838	-0.51276	-1.13837	-0.51276
1.00	−1.32503	−0.83709	−1.32503	−0.83710
2.00	−1.65235	−1.28726	−1.65235	−1.28726

Table 3
comparison of present results with previous study at ϕ = 0,M = 1, λ = 0.5..

	$Nu/\sqrt{Re}$	
	Mustafa et al. [22]	present results
Rd =0,θw=1	1.66171	1.66171
Rd =1,θw=1	2.15308	2.15308
Rd =1,θw=1.2	2.23565	2.23565

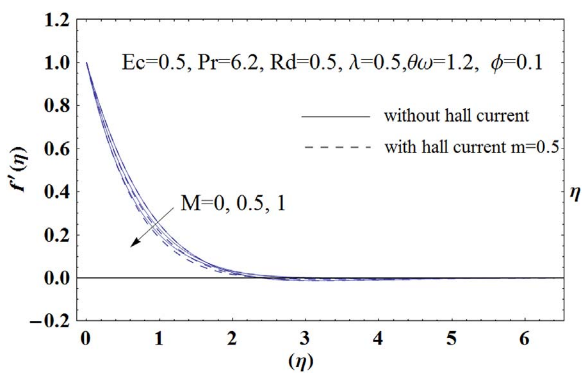

Fig. 2. Longitudinal velocity profile with increasing of Hartman number (weak magnetic strength).

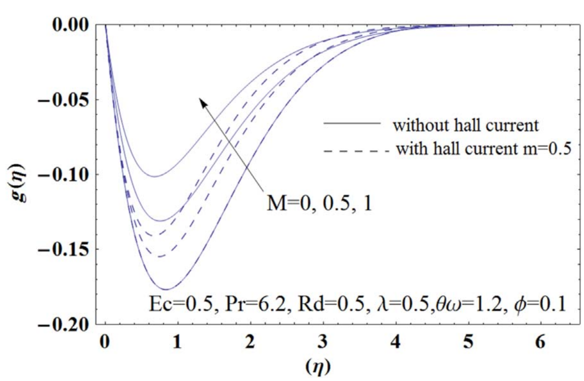

Fig. 3. Transversal velocity profile with increasing of Hartman number (weak magnetic strength).

other side, the presence of hall current beside the magnetic field indicated by dash line for each figure. Obviously, the Hall current has no clear effect on the longitudinal velocity unlike the transversal velocity, which increased in the presence of hall current. Moreover, the increasing of the transversal velocity is clearer in the case of strong magnetic strength as shown in Fig. 4. Physically, the presence of hall current beside the magnetic field generates a drag normal force called a Lorentz force, this force perpendicular to the main direction of the flow and magnetic field lines, which means that the force direction would be along y-axis. Therefore, one can say that the increasing of the transversal velocity resulted from the Lorentz force

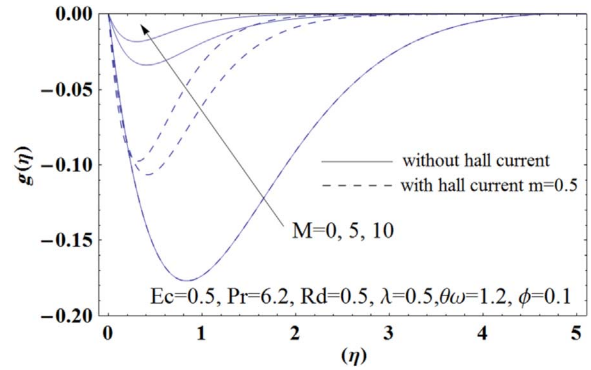

Fig. 4. Transversal velocity profile with increasing of Hartman number (strong magnetic

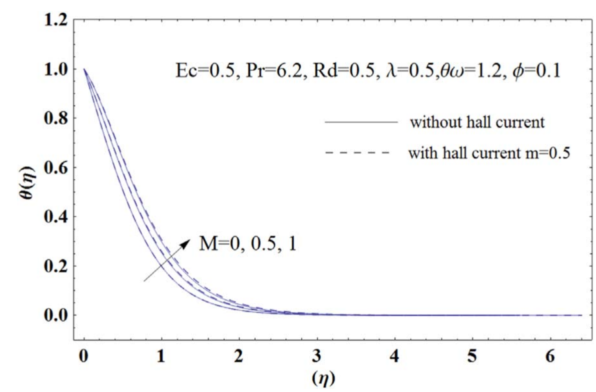

Fig. 5. Temperature profile with increasing of Hartman number (weak magnetic strength).

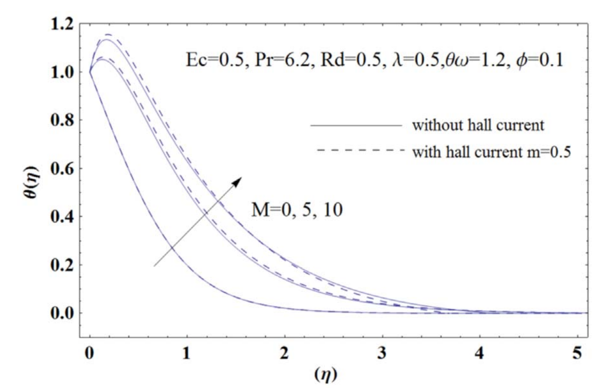

Fig. 6. Temperature profile with increasing of Hartman number (strong magnetic strength).

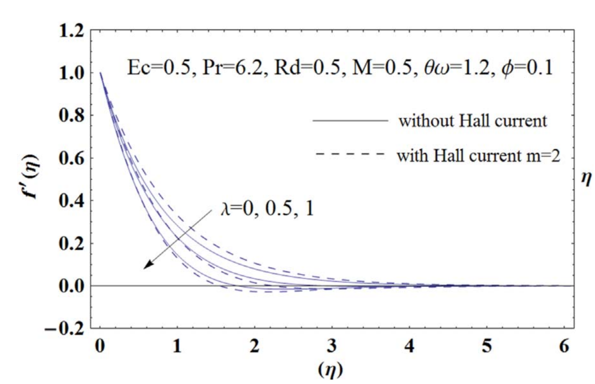

Fig. 7. Longitudinal velocity profile with increasing of rotation parameter.

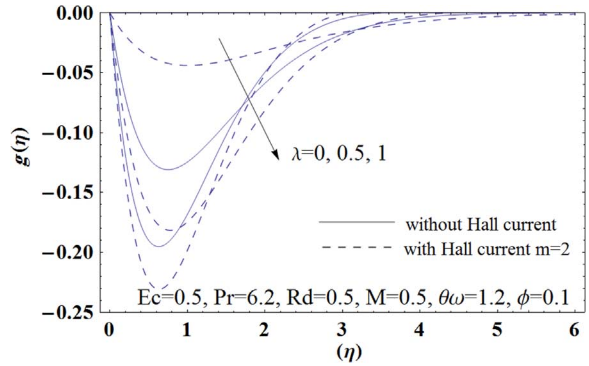

Fig. 8. Transversal velocity profile with increasing of rotation parameter.

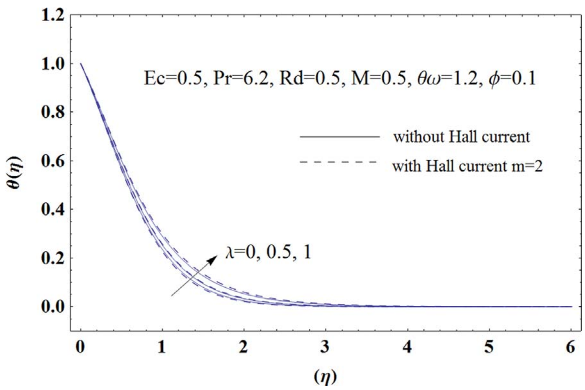

Fig. 9. Temperature profile with increasing of rotation parameter.

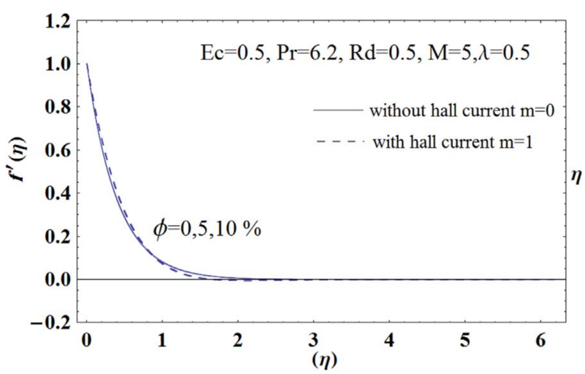

Fig. 10. Longitudinal velocity profile with increasing of nanoparticle volume fraction.

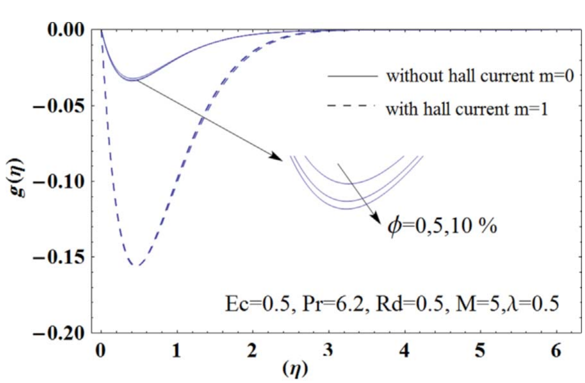

Fig. 11. Transversal velocity profile with increasing of nanoparticle volume fraction.

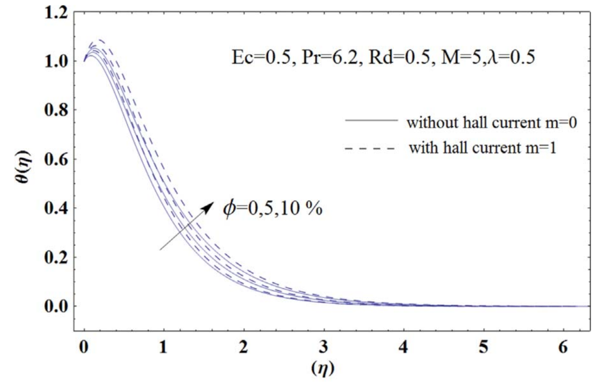

Fig. 12. Temperature profile with increasing of nanoparticle volume fraction.

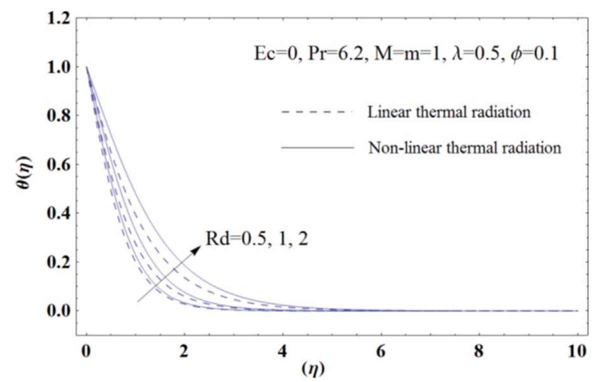

Fig. 13. Temperature profile with increasing of radiation parameter.

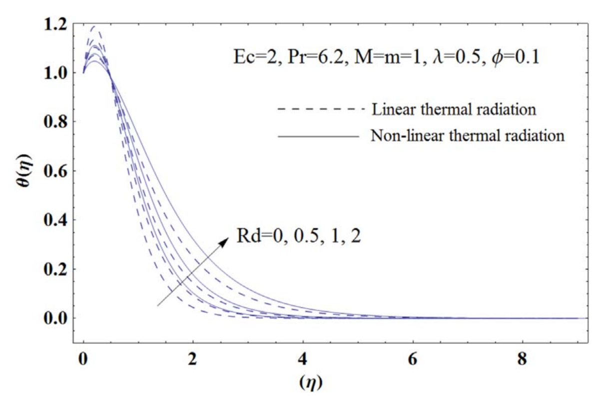

Fig. 14. Temperature profile with increasing of radiation parameter.

which forcing the fluid molecules to change the path direction.

The effect of magnetic field on the boundary layer temperature profiles presented by Figs. 5 and 6. Generally, the presence of magnetic field raises the motion of the fluid molecules near the surface, which means increasing of the boundary layer temperature. Moreover, Fig. 6 indicates that in the presence of strong magnetic strength the temperature of the boundary layer exceeding the surface temperature to critical point before decaying to the ambient temperatureT_∞ at the end of the boundary layer. In addition, the presence of hall current increases the boundary layer temperature as well as the thermal boundary layer thickness, this phenomenon occurs due to the effect joule heating which resulting from the strong magnetic field.

- **The effect of rotation parameter**

Rotation parameter (λ) is the ratio between the rotation rate and

Table 4
values of velocity gradient and temperature gradient at the surface at $Rd = Ec = \lambda = 0.5$, $\theta w = 1.2$..

ϕ	m	M	$f''(0)$	$g'(0)$	$\theta'(0)$	$C_{fx}\sqrt{Re}$	$C_{fy}\sqrt{Re}$	$Nu/\sqrt{Re}$
0	0	0	−1.138381	−0.51276	−1.156583	−1.138381	−0.51276	2.1558711
		0.5	−1.304121	−0.429593	−0.885799	−1.304121	−0.429593	1.6511303
		1	−1.465502	−0.372366	−0.653805	−1.465502	−0.372366	1.2186927
	2	0	−1.138381	−0.51276	−1.156583	−1.138381	−0.51276	2.1558711
		0.5	−1.206977	−0.56815	−0.879404	−1.206977	−0.56815	1.6392096
		1	−1.271961	−0.618957	−0.620739	−1.271961	−0.618957	1.1570584
0.1	0	0	−1.188939	−0.535533	−1.042855	−1.547225	−0.696916	2.2276842
		0.5	−1.351556	−0.453064	−0.734457	−1.758846	−0.589595	1.5689026
		1	−1.510612	−0.395076	−0.470144	−1.965833	−0.514131	1.0042938
	2	0	−1.188939	−0.535533	−1.042855	−1.547225	−0.696916	2.2276842
		0.5	−1.256345	−0.590018	−0.723805	−1.634943	−0.76782	1.5461496
		1	−1.320391	−0.640221	−0.42566	−1.718289	−0.83315	0.9092693

stretching rate. The influence of this parameter on the longitudinal and transversal velocities had shown through Figs. 7 and 8. As expected, increase the rotation rate with respect to stretching rate leads to decreasing in the longitudinal velocity and increasing in the transversal velocity. Moreover, one can observe from Fig. 7 that the presence of hall current increases the longitudinal velocity only in the absence of rotation and the opposite appears under the effect of rotation. Physically, this occurs due to the both effect of Lorentz force and rotational motion whose forced the fluid to move in the transversal direction. On the other hand, Fig. 8 confirms that the hall current has a direct and clear effect on the transversal velocity, such that for no rotation there is no transverse velocity due to the absence of the angular velocity. Nevertheless, the profile of the velocity indicates that the presence of hall current generate a transverse velocity in spite of the lack of rotation of the fluid. Fig. 9 depicts the effect of rotation parameter on the temperature profile. it is observed that the temperature rose in the presence of rotation parameter due to the increasing of the kinetic energy of the fluid. In addition, one can observe that the hall current has a limited effect on the temperature profile due to weak magnetic strength.

- **The effect of nanoparticles volume fraction**

Nanoparticle volume fraction or nanoparticles concentration is the ratio that controls the density, viscosity, the thermal conductivity of the fluid. The influence of this ratio on the velocities profiles sketched in Figs. 10 and 11. Clearly, there is no active effect of this ratio on the longitudinal velocity and limited effect on the transverse velocity such that Fig. 11 indicate that as the nanoparticles concentration enlarges the velocity increases. Fig. 12 depicts the variation of the temperature under the increasing of nanoparticles concentration. The variation indicates, as the thermal conductivity and the viscous force increase, the boundary layer temperature increases. Moreover, the presence of hall current increases the temperature especially near the surface.

- **The effect of nonlinear thermal radiation**

Table 6
values of temperature gradient at the surface at $Pr = 6.2$, $\phi = 0.1$, $\lambda = M = m = 0.5$.

Ec	θw	Rd	$\theta'(0)$	$Nu/\sqrt{Re}$	Ec	$\theta'(0)$	$Nu/\sqrt{Re}$
0	1	0.00	−1.41343	1.79808	2	2.0541	−2.61308
		0.50	−1.14178	2.02340		1.4677	−2.60099
		1.00	−0.96559	2.19396		1.1365	−2.58230
	1.2	0.00	−1.41343	1.79808		2.0541	−2.61308
		0.50	−0.990818	2.11653		1.216	−2.59756
		1.00	−0.778965	2.33700		0.8554	−2.56624

Figs. 13 and 14 show the effect thermal radiation with/without the presence of Eckert number on the boundary layer temperature. It is worth mentioning that, Eckert number is the ratio between the flow's kinetic energy and the enthalpy. In addition, the presence of Eckert number beside the presence of magnetic field means that the boundary layer subjected to joule heating. In general, the boundary layer that subjected to thermal radiation has a large temperature comparing with the absence of it. On the other hand, Fig. 13 shows a smooth decay of the temperature profile to zero directly in the absence of the Eckert number and an exponentially profile for the presence of it, such that the temperature increases to its maximum values near the surface before decaying to zero far from it.

The influence of all embedded parameters on the surface shear stress and heat flux express through the study of the dimensionless skin friction and local Nusselt number. Table 4 presents the values of these physical quantities for regular fluid ($\phi = 0$)and nanofluids with 10% nanoparticles concentration, it is clear that the MHD boundary layer has high skin friction coefficientsC_{fx} & C_{fy} and Nusselt numberNu comparing with the boundary layer that Not exposed to a magnetic field. Moreover, it is clear that the skin friction increases in each direction by 30~40% and the Nusselt number decreases by 5~20% in the case of 10% nanoparticles concentration comparing with the pure fluid. In addition, Table 4 displays that the presence of magnetic field

Table 5
values of velocity gradient and temperature gradient at the surface at =6.2, $\phi = 0.1$ $Rd = Ec = M = 0.5$, $\theta w = 1.2$.

m	λ	$f''(0)$	$g'(0)$	$\theta'(0)$	$C_{fx}\sqrt{Re}$	$C_{fy}\sqrt{Re}$	$Nu/\sqrt{Re}$
0	0.00	−1.26613	0.00000	−0.75499	−1.64767	0.00000	1.61276
	0.50	−1.35156	−0.45306	−0.73446	−1.75885	−0.58959	1.56890
	1.00	−1.50612	−0.79040	−0.69247	−1.95998	−1.02858	1.47920
0.5	0.00	−1.099328	−0.113994	−0.755949	−1.43061	−0.14835	1.61481
	0.50	−1.256345	−0.590018	−0.723805	−1.63494	−0.76782	1.54615
	1.00	−1.441862	−0.909529	−0.676352	−1.87637	−1.18361	1.44478

beside the hall current increase the x-direction skin friction by 7~15% and decrease the y-direction values by 25~60% and increase the local Nusselt number by 2~9%. On the other hand, Table 5 shows the effect of rotation ratio on dimensionless physical quantities in the presence/absence of hall current, generally, the increasing of rotation ratio increases the skin friction in each direction and decreases the Nusselt number. Moreover, evident from the table that the Hall current has a direct impact on the transversal friction on the surface even in the absence of the fluid rotation.

Table 6 shows the values of the local Nusselt number with variation of thermal radiation parameter and temperature ratio in the presence/absence of joule heating (Eckert number). For the first instant, one can observe that the values of Nusselt number are positive for no joule heating effects and negative values for the presence of it. It worth mentioning that, the sign of Nusselt number indicates to the path of the heat flux from the surface to the boundary layer or the reverse. So on can conclude that the presence of joule heating changing the path of the heat flux to be from the boundary layer to the surface, that Explains the behavior of the temperature profile which plotted in Fig. 9. On the other hand, the value of Nusselt number increases in under the effect of thermal radiation parameter and temperature ratio only if the Eckert number absences and the opposite is true in the presence of Eckert number.

6. Conclusions

The effect of nonlinear thermal radiation and hall current with joule heating on the MHD rotating flow over a stretching plate investigated and the following results obtained:

- The existence of the hall current increases the transverse velocity, as well as has a limited impact on the longitudinal velocity.
- Boundary layer temperature exposed to nonlinear thermal radiation is higher than that exposed to linear thermal radiation.
- Joule heating increases the boundary layer temperature near the surface especially in the linear model of thermal radiation.
- Magnetic field with hall current increases the longitudinal skin friction and reduces the transversal skin friction.
- Direction of heat flux flow changes in the presence of MHD flow with joule heating.
- Simplify the modeling by using linear thermal radiation model, gives lower values for the rate of heat transfer.

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